RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2015-18]

B.A./B.Sc. FIRST SEMESTER (July – December) 2015

Mid-Semester Examination, September 2015

Date : 14/09/2015

PHYSICS (Honours)

Time : 11 am – 1 pm

Paper : I

Full Marks : 50

[Use a separate Answer Book for each group]

[Answer <u>five</u> questions taking <u>atleast one</u> from each group]

Group – A

1. a) Consider the matrix $B=\begin{pmatrix} 0 & -i & i \\ i & 0 & -i \\ -i & i & 0 \end{pmatrix}$

Check whether B is (i) real, (ii) symmetric, (iii) antisymmetric, (iv) singular, (v) orthogonal, (vi) Hermitian, (vii) anti-Hermitian, (viii) unitary. [4]

b) Given the matrix, $A=\begin{pmatrix} 1 & \alpha & 0 \\ \beta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ where α and β are nonzero complex numbers. Find the eigenvalues and eigenvectors of A. Find the respective conditions for (i) the eigenvalues to be real and (ii) the eigenvectors to be orthogonal. [4+2]

2. a) Consider the Differential Equation : $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=-2\cos h\,x$ Find y when y = 0 and $\frac{dy}{dx}=1$ at x = 0. [4]

b) Find the series solution of the Harmite equation $\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2\alpha y=0$. [6]

Group – B

3. a) What is the geometrical interpretation of gradient of a function? [2]

b) Show that $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{F})=0$. [2]

c) What is a conservative force field?

If $\vec{v}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is a conservative force field find a, b, c also find the scalar potential of the vector field. [1+2+3]

4. a) Write Gauss's divergence theorem. [2]

b) Verify Green's theorem in the plane for $\oint_C (xy+y^2)dx+x^2dy$, where C is the closed curve of the region bounded by y = x and y = x^2. [4]

c) Verify Stoke's theorem for $\vec{A}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$, considering upper half surface of sphere $x^2+y^2+z^2=1$. [4]

Group – C

5. a) Define inertial frames of reference (IFR) and obtain the Galilean transformation rules, connecting the space and time coordinates in two IFRs, S and S´ in uniform relative motion. State clearly all the assumptions made. [4]

b) Use (a) to obtain the transformation rules for the linear momentum $\vec{p}$, and kinetic energy T of a particle of mass m. [4]

c) Show that the relation $T = \frac{p^2}{2m}$ is Galilean invariant. [2]

6. a) Obtain expressions for the velocity and acceleration of a particle in plain polar coordinates (r, θ). [6]

b) Given that $r = ct$, $\theta = \Omega t$, where c and Ω are positive constants, find at time t, the (i) velocity vector, $\vec{v}$ (ii) acceleration vector, $\vec{a}$ (iii) speed of the particle, $v = |\vec{v}|$. Deduce that for t > 0, the angle between $\vec{v}$ and $\vec{a}$ is always acute. [4]

Group – D

7. a) State Fermat's principle. Using this principle show that all rays passing through the focus of a parabolic reflector will be rendered parallel to the axis after reflection. [1+2]

b) Establish the thin lens formula by applying Fermat's principle. [4]

c) What is spherical aberration? Explain how it can be minimised by using two lenses separated by a distance. [1+2]

8. a) Show that the distance between the two principal points is equal to the distance between the two nodal points. [2]

b) Find out the system matrix for a combination of two thin lenses separated by a distance. Hence determine the equivalent focal length and the principal points. [4]

c) A sphere of glass (n = 1·5) is of 20 cm diameter. A parallel beam of light enters from one side. Determine the system matrix and find where the beam will get focussed on the other side. [4]`

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